ON LUSTERNIK-SCHNIRELMANN CATEGORY OF SO(10)

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Abstract: Let G be a compact connected Lie group and $p : E \to \Sigma A$ a principal G-bundle whose characteristic map is denoted by $\alpha : A \to G$. We assume that $clA \leq 1$ and $clG \leq m$: let $A = \Sigma A_0$ for a space A_0 and $\{K_i{\to}F_{i-1}{\hookrightarrow}F_i \,|\, 1{\leq}i{\leq}n,\ F_0{=}\{*\}\ F_1{=}\Sigma K_1 \text{ and } F_n{\simeq}G\}$ be a cone-decomposition of G of length m. Our main result is as follows: If there is a subspace $F'_1 \subset F_1$ such that $F'_1 = \Sigma K'_1$, $K'_1 \subset K_1$ and $F_i F'_1 \subset F_{i+1}$ up to homotopy for any i, we have $\mathrm{cat}X \leq m{+}1$, if firstly the characteristic map α is compressible into F'_1, secondly the Berstein-Hilton Hopf invariant $H_1(\alpha)$ vanishes in $[A, \Omega F'_1{*}\Omega F'_1]$ and thirdly K_m is a sphere. We apply this to the principal bundle $\mathrm{SO}(9) \hookrightarrow \mathrm{SO}(10) \to S^9$ to determine $\mathrm{cat}\mathrm{SO}(10)$.