On holomorphic Lefschetz number of the Reeb flow of toric Sasakian manifolds

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Lefschetz number of the Reeb flow

$X = \mathbb{C}^n \setminus \{0\}$

Consider a S^1-action ρ on X given by

$$q \cdot (z_1, \dots, z_n) = (qz_1, \dots, qz_n) \quad (q \in S^1).$$

Problem

Compute the holomorphic Lefschetz number of $q \in S^1$:

$$L(q, X) = \sum_{i=0}^{n} (-1)^i \operatorname{Trace}(q^* : H^{0,i}(X) \to H^{0,i}(X)).$$

It is well known that

$$H^{0,i}(X) = \begin{cases} \mathcal{O}(X) & i = 0, \\ 0 & i > 0. \end{cases}$$

Lefschetz number of the Reeb flow

Consider $\oplus_k \mathcal{O}_k(X)$ in the place of $\mathcal{O}(X)$, where

$$\mathcal{O}_k(X) = \{h \in \mathcal{O}(X) \mid h(qx) = q^k h(x)\}.$$

Since $\dim \mathcal{O}_k(X) = \frac{(n+k-1)!}{(n-1)!k!}$, we get

$$L(q,X) = \sum_{k=0}^{\infty} q^k \dim \mathcal{O}_k(X) =$$

$$\sum_{k=0}^{\infty} q^k \frac{(n+k-1)!}{(n-1)!k!} = \Big(\frac{1}{(n-1)!} \sum_{k=0}^{\infty} q^{n+k-1}\Big)^{(n-1)} .$$

$L(q,X)$ may not be well-defined on S^1.

(M, g) : a (connected compact) Riemannian manifold

η : a contact 1-form on M

Proposition

(M, g, η) is a Sasakian manifold iff its metric cone $(M \times \mathbb{R}_{>0}, r^2 g + dr \otimes dr, d(r^2\eta))$ is a Kähler manifold.

Introduction

Example

- S^{2n-1} whose cone is $S^{2n-1} \times \mathbb{R}_+ \cong \mathbb{C}^n \setminus \{0\}$
- positive S^1-bundle over Kähler manifolds
  whose cone is the associated $\mathbb{C}^\times$-bundle
- the links of certain isolated singularities of complex varieties
- contact toric manifolds of Reeb type

Introduction

Sasaki-Einstein manifolds have been studied with motivation in

- the $\mathrm{AdS}_5/\mathrm{CFT}_4$ correspondence and
- construction of Einstein metrics.

Some conjectures by physicists remain open.

c.f. D. Martelli, J. Sparks and S.-T. Yau,
Sasaki-Einstein manifolds and volume minimisation,
Comm. Math. Phys. **280** (2008), no. 3, 611–673.

Conjectures

Theorem (Martelli-Sparks-Yau)

For a closed Sasaki-Einstein manifold M^{2n-1}, the volume is an algebraic integer.

Conjecture (Martelli-Sparks-Yau)

The degree of the volume of a closed SE manifold M^{2n-1} is equal to $(n-1)^{\operatorname{rank} M - 1}$.

Conjecture (Akishi Kato)

$\mathcal{S}$: the set of isometric classes of toric SE 5-mfds with hol trivial κ_X.
The volume map $\mathcal{S} \to \mathbb{R}; M \mapsto \mathrm{vol}(M)$ is injective.

Introduction

(M^{2n-1}, g, η) : a closed Sasakian manifold

ξ : the Reeb vector field of η defined by $\iota_\xi d\eta = 0$ and $\eta(\xi) = 1$.

The flow generated by ξ is called the *Reeb flow* of η.

Lemma

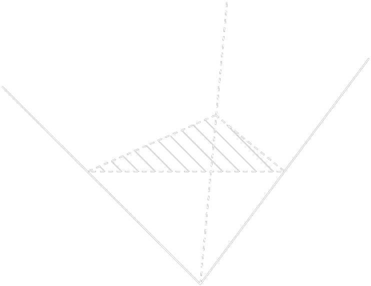
The closure T of the Reeb flow in $\mathrm{Isom}(M, g)$ is a torus.

Consider the toric case: $\dim T = n$.

Now the momentum polytope Δ of such a Sasakian manifold is the image of the contact moment map:

$$\begin{array}{cccc} \Psi : & M & \longrightarrow & \mathrm{Lie}(T)^* \\ & x & \longmapsto & (X \mapsto \eta(X_{\#})(x)), \end{array}$$

where $X_{\#}$ is the fundamental vector field of $X \in \mathrm{Lie}(T)$.



Introduction

$S^{2n-1} \subset \mathbb{R}^{2n}$: the unit sphere
$\eta_{\mathrm{std}} = \sum_{i=1}^{n}(y_i dx_i - x_i dy_i)$: the standard contact form on S^{2n-1}
$b = (b_1, \ldots, b_n) \in (\mathbb{R}_{>0})^n$

Consider

$$\eta_b = \frac{\eta_{\mathrm{std}}}{\sum_{i=1}^{n} b_i(x_i^2 + y_i^2)} \in \Omega^1(S^{2n+1}).$$

Here S^{2n-1} admits a Sasakian structure (η_b, g_b), where the metric g_b is determined by η_b and the standard CR structure on S^{2n-1}.

The Reeb vector field ξ_b of η_b is

$$\xi_b = \sum_{i=1}^{n} b_i \left(y_i \frac{\partial}{\partial x_i} - x_i \frac{\partial}{\partial y_i} \right).$$

Martelli-Sparks-Yau's theorems

M^{2n+1} : a closed manifold,
$\mathcal{S}$: the space of Sasakian metrics on M.

$$\begin{array}{rccc} \mathrm{Vol}: & \mathcal{S} & \longrightarrow & \mathbb{R} \\ & g & \longmapsto & \mathrm{Vol}(M,g) \end{array}$$

It is easy to see that $\mathrm{Vol}(M,g) = \frac{1}{2^n n!}\int_M \eta \wedge (d\eta)^n$.

Proposition (Martelli-Sparks-Yau)

For Sasakian manifolds whose cone admits holomorphically trivial canonical line bundle, Vol *is equal to the Einstein-Hilbert action up to a constant on* $\mathcal{S}$*.*

In particular, Sasaki-Einstein metrics are critical points of Vol*.*

Martelli-Sparks-Yau's theorems

$q \in T \subset \mathrm{Aut}(M, g, \eta)$

$X = M \times \mathbb{R}_+$: the cone of M

The holomorphic Lefschetz number $L(q)$ should be defined by

$$L(q) = \sum_{i=0}^{n} (-1)^i \operatorname{trace}\left(q : H^{0,i}(X) \to H^{0,i}(X)\right),$$

Since

$$H^{0,i}(X) \cong \begin{cases} \mathcal{O}(X) & i = 0, \\ \{0\} & i > 0. \end{cases}$$

Hence

$$L(q) = \operatorname{trace}\left(q : \mathcal{O}(X) \to \mathcal{O}(X)\right).$$

Martelli-Sparks-Yau's theorems

Assume the well-definedness of $L(q)$ to have a function L on T.

This L should have a pole at $1 \in T$ by

$$L(1) = \dim \mathcal{O}(X) = \infty.$$

Theorem (Martelli-Sparks-Yau)

Take $b \in \mathrm{Lie}(T)$ so that $b_{\#} = \xi$. Then we have

$$\mathrm{Vol}(M) = \frac{2\pi^n}{(n-1)!} \lim_{t \to 0} \ t^n \ L(\exp(-tb)),$$

Main result

Theorem

(M^{2n-1}, g, η) : *a closed Sasakian manifold* $(n > 1)$,

$X = M \times \mathbb{R}_{>0}$

Assume that

1. *an* n*-dim torus* $T \subset \mathrm{Aut}(M, g, \eta)$ *contains the Reeb flow, and*
2. κ_X *is holomorphically trivial.*

Let $T_{\mathbb{C}}$ *be the complexification of* T, *which acts on* X.

Then $L(q)$ *is a well-defined holomorphic fcn on* $\{ q \in T_{\mathbb{C}} \mid |q| \ll 1 \}$.

Main result

Definition

$\mathcal{H}$: a separable Hilbert space, $\varphi : \mathcal{H} \to \mathcal{H}$ bounded

φ is *of trace class* if the series

$$\sum_i \langle (\varphi^* \varphi)^{1/2} e_i, e_i \rangle$$

absolutely converges for some orthonormal basis $\{e_i\}$ of $\mathcal{H}$.

We will complete $\mathcal{O}(X)$ as a Hilbert space.

Remark

If $X = \mathbb{C}^2 \setminus \{0\}$, for $q \in \mathbb{C}^\times$ with $|q| > 1$, for any completion $\mathcal{H}$ of $\mathcal{O}(X)$, the extention of q^* to $\mathcal{H} \to \mathcal{H}$ is not bounded, because the set of the eigenvalues of q^* is not bounded.

Main result

Take a principal T-orbit Σ in X.
$\mathcal{M}(\Sigma, \mathbb{C})$: the space of Lebesgue measurable fcns on Σ
The restriction map $\rho : \mathcal{O}(X) \longrightarrow \mathcal{M}(\Sigma, \mathbb{C})$ is injective.
Consider an inner product $\langle \cdot, \cdot \rangle$ on $\mathcal{M}(\Sigma, \mathbb{C})$ given by

$$\langle f, g \rangle = \int_{\Sigma} f \overline{g} d\,\mathrm{vol}_{\Sigma}, \qquad f, g \in \mathcal{M}(\Sigma, \mathbb{C}).$$

Take the completion with this inner product

$$\mathcal{H} = \overline{\rho(\mathcal{O}(X))}.$$

Let $\mathcal{S} = \mathcal{C}^* \cap (\mathfrak{t}_{\mathbb{Z}})^*$, where $\mathcal{C}^*$ is the moment polytope of $M \times \mathbb{R}_+$

$\mathcal{O}(X)$ consists of convergent power series of polynomials z^m for $m \in \mathcal{S}$.

$\mathcal{H}$ has an orthonormal basis $\{\frac{1}{\|z^m\|} z^m\}_{m \in \mathcal{S}}$.

For $q \in T_{\mathbb{C}}$, extend $q : \mathcal{O}(X) \to \mathcal{O}(X)$ to $q : \mathcal{H} \to \mathcal{H}$ by the linearity.

Proposition

Let $q \in T_{\mathbb{C}}$. If $|q| \ll 1$, then $q : \mathcal{H} \to \mathcal{H}$ is bounded and of trace class.

Proof.

Let $\mathcal{C}^*_{\mathrm{std}} = (\mathbb{R}_{\geq 0})^n$. We can assume that $\mathcal{C}^* \subset \mathcal{C}^*_{\mathrm{std}}$. It is easy to see that $q^* = \overline{q}$. Let $\hat{q} = (|q_1|, \ldots, |q_n|)$. Then we have

$$\sum_{m \in \mathcal{S}} \left\langle (q^* q)^{1/2} \frac{1}{\|z^m\|} z^m, \frac{1}{\|z^m\|} z^m \right\rangle = \sum_{m \in \mathcal{S}} \hat{q}^m.$$

Since $\mathcal{C}^* \subset \mathcal{C}^*_{\mathrm{std}}$, we have $\sum_{m \in \mathcal{S}} \hat{q}^m \leq \sum_{m \in (\mathbb{Z}_{\geq 0})^n} \hat{q}^m$. By assumption, we have

$$\sum_{m \in (\mathbb{Z}_{\geq 0})^n} \hat{q}^m = \prod_{i=1}^{n} \frac{1}{1 - |q_i|}.$$

□

Consider a function $F : \mathrm{Lie}(T_{\mathbb{C}}) \to \mathbb{C} \cup \{\infty\}$ defined by

$$F(b) = \frac{2\pi^n}{(n-1)!} \int_{\mathcal{C}^*} e^{-(b,y)} dy_1 \cdots dy_n,$$

where a coordinate $(y_1, \ldots, y_n)$ on $\mathfrak{t}^*$ associated with the fixed integral basis. Here $(\cdot, \cdot)$ is the canonical pairing between $\mathfrak{t}$ and $\mathfrak{t}^*$.

Theorem (Martelli-Sparks-Yau)

For each b in $\mathcal{C}$, we have

$$F(b) = \mathrm{Vol}(M, g_b),$$

where g_b is the Sasakian metric obtained by deformation of type I whose Reeb vector field is equal to $b_{\#}$.

Let $\omega = \frac{d(r^2\eta)}{2}$ be the symplectic form on X. By Stokes theorem, we have

$$\mathrm{vol}(M) = \frac{1}{2^{n-1}} \int_M \eta \wedge \frac{(d\eta)^{n-1}}{(n-1)!} = 2n \int_{X_{\leq 1}} \frac{\omega^n}{n!},$$

where $X_{\leq 1} = \cup_{0<r\leq 1} M \times \{r\}$. By integrating along the fibers of $r : X \to \mathbb{R}$ and using $\int_0^\infty r^{2n-1} e^{-r^2/2} dr = 2^{n-1}(n-1)!$, we have

$$2^n n! \int_{X_{\leq 1}} \omega^n = \int_X e^{-r^2/2} \omega^n.$$

Then it follows that

$$\mathrm{vol}(M) = \frac{1}{2^{n-1}(n-1)!} \int_X e^{-r^2/2} \frac{\omega^n}{n!}.$$

$(\phi_1, \ldots, \phi_n) : \mathfrak{t} \to \mathbb{R}^n/2\pi\mathbb{Z}^n$: the coordinate on $\mathfrak{t}$ correspond to an integral basis of $\mathfrak{t}_{\mathbb{Z}}$.

$(y_1, \ldots, y_n)$: the coordinate on $\mathfrak{t}^*$ which corresponds to the dual basis.

Since we have $\omega = \sum_{i=1}^n dy_i \wedge d\phi_i$ on $\Psi^{-1}\big(\operatorname{int}(\mathcal{C}^*)\big)$, by integrating along the torus fibers of Ψ, we get

$$\begin{aligned}
&\frac{1}{2^{n-1}(n-1)!}\int_X e^{-r^2/2}\frac{\omega^n}{n!} \\
=&\frac{1}{2^{n-1}(n-1)!}\int_X e^{-r^2/2}|d\phi_1\cdots d\phi_n dy_1\cdots dy_n| \\
=&\frac{2\pi^n}{(n-1)!}\int_{\mathcal{C}^*} e^{-r^2/2}dy_1\cdots dy_n.
\end{aligned}$$

Here $r^2/2$ is the Hamiltonian function of ξ, namely, $-\big(b, \Psi(p)\big) = r^2/2$. Thus, we have

$$\operatorname{vol}(M) = \frac{2\pi^n}{(n-1)!}\int_{\mathcal{C}^*} e^{-(b,y)}dy_1\cdots dy_n = F(b).$$

Corollary

We have

$$F(b) = \frac{2\pi^n}{(n-1)!} \lim_{t\to 0} t^n L(e^{-bt})$$

for b in a domain $\{ b \in \mathrm{Lie}(T_{\mathbb{C}}) \mid \mathrm{Im}\, b \gg 0 \}$.

For $q \in T_{\mathbb{C}}$, we have

$$L(q) = \sum_{m\in\mathcal{S}} q^m.$$

Thus,

$$L(e^{-bt}) = \sum_{m\in\mathcal{S}} e^{-(b,m)t}.$$

For b with $\mathrm{Im} \gg 0$, the right hand side is well defined. By the definition of Riemann integral, we have

$$\lim_{t\to 0} t^n L(e^{-bt}) = \lim_{t\to 0} t^n \sum_{m\in\mathcal{S}} e^{-(b,m)t} = \int_{\mathcal{C}^*} e^{-(b,y)} dy_1 \cdots dy_n = F(b).$$

Corollary

(M^{2n-1}, g, η) : *a closed Sasakian manifold* $(n > 1)$,

$X = M \times \mathbb{R}_{>0}$

Assume that

- *an* n*-dim torus* $T \subset \mathrm{Aut}(M, g, \eta)$ *contains the Reeb flow, and*
- κ_X *is holomorphically trivial.*

1.
$$\frac{2\pi^n}{(n-1)!} \lim_{t \to 0} t^n L(e^{-bt}) = \frac{2\pi^n}{(n-1)!} \int_{\mathcal{C}^*} e^{-(b,y)} dy_1 \cdots dy_n$$
for $b \in \mathrm{Lie}(T_{\mathbb{C}})$ *with* $\mathrm{Im}\, b \gg 0$.

2.
$$\mathrm{Vol}(M, g_b) = \frac{2\pi^n}{(n-1)!} \int_{\mathcal{C}^*} e^{-(b,y)} dy_1 \cdots dy_n,$$
where g_b *is the Sasakian metric obtained by deformation of type I whose Reeb vector field is equal to* $b_{\#}$.

Example

Consider a cone $\mathcal{C}^* = \{ y \in \mathbb{R}^3 \mid (v_i, y) \geq 0 \}$, where v_1, v_2, $v_3 \in \mathbb{R}^3$ are given by

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}.$$

Here $\mathcal{C}^*$ is the moment polytope of the cone of a 5-dimensional toric Sasakian manifold M (Cho-Futaki-Ono's characterization).

Example

The vectors tangent to 1-dimensional faces of $\mathcal{C}^*$ are

$$w_1 = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}, \quad w_2 = \begin{pmatrix} 0 \\ 4 \\ -2 \end{pmatrix}, \quad w_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}.$$

Here note that we have

$$(x, w_i) = \det(x, v_j, v_k), \qquad \forall x \in \mathbb{R}^3,$$

where $(i, j, k) = (1, 2, 3)$, $(2, 3, 1)$, $(3, 1, 2)$.

Example

We will compute the characteristic function $\sigma_{\mathcal{C}^*}$ of $\mathcal{C}^*$ defined by

$$\sigma_{\mathcal{C}^*}(y) = \sum_{m \in \mathcal{S}} y^m,$$

where $\mathcal{S} = \mathcal{C}^* \cap \mathbb{Z}^3$, with the technique of Beck-Haase-Sottile. Then, the Martelli-Sparks-Yau formula gives the volume of M.

Example

Consider the parallelepiped $\mathcal{P}$ spanned by w_1, w_2, w_3, i.e.,

$$\mathcal{P} = \{ \, y \in \mathbb{R}^3 \mid y = c_1 w_1 + c_2 w_2 + c_3 w_3, \; 0 \leq \exists c_i < 1 \; (i = 1, 2, 3) \, \}.$$

Since $\mathcal{C}^*$ is tiled with translates of $\mathcal{P}$ by a semigroup

$$\mathbb{Z}_{\geq 0} w_1 \oplus \mathbb{Z}_{\geq 0} w_2 \oplus \mathbb{Z}_{\geq 0} w_3,$$

for $y \in \mathbb{C}^3$ with sufficiently small absolute value, we have

$$\sigma_{\mathcal{C}^*}(y) = \frac{\sigma_{\mathcal{P}}(y)}{(1 - y^{w_1})(1 - y^{w_2})(1 - y^{w_3})}.$$

Example

Let $u_1 = (1, -2, 1)^T$ and $u_2 = w_2/2$. It is easy to see that the integer points contained in $\mathcal{P}$ is 0, u_1, u_2 and $u_1 + u_2$. Then we have

$$\sigma_{\mathcal{P}}(y) = 1 + y^{u_1} + y^{u_2} + y^{u_1+u_2}$$

and hence

$$\sigma_{\mathcal{C}^*}(y) = \frac{1 + y^{u_1} + y^{u_2} + y^{u_1+u_2}}{(1 - y^{w_1})(1 - y^{w_2})(1 - y^{w_3})}.$$

Example

Let $L(q)$ be the holomorphic Lefschetz number of $q \in (\mathbb{C}^\times)^3$. Since $L(q) = \sigma_{\mathcal{C}^*}(q)$ as we saw in the last section, take $b = (b_1, b_2, b_3)^T \in \mathbb{C}^3$ and substitute $y = e^{-bt} = (e^{-b_1 t}, e^{-b_2 t}, e^{-b_3 t})^T$ to the last equation to have

$$L(e^{-bt}) = \frac{1 + e^{-(b,u_1)t} + e^{-(b,u_2)t} + e^{-(b,u_1+u_2)t}}{(1 - e^{-(b,w_1)t})(1 - e^{-(b,w_2)t})(1 - e^{-(b,w_3)t})}.$$

Thus we have

$$\lim_{t \to 0} t^3 L(e^{-bt}) = \frac{4}{(b, w_1)(b, w_2)(b, w_3)}.$$

By the formula of Martelli-Sparks-Yau, we have

$$\mathrm{vol}(M) = \frac{4\pi^3}{(b, w_1)(b, w_2)(b, w_3)}.$$

Example

The volume of toric Sasakian manifolds M can be computed in four other ways:

- $\mathrm{Vol}(M) = C\,\mathrm{Vol}(\Delta)$ by Martelli-Sparks-Yau. Then Lawrence's formula of the volume of polytope can be used.
- MSY's localization formula of the volume of M on an equivariant resolution of the singularity at the origin of $M \times \mathbb{R}_+$.
- the localization formula of basic cohomology of Killing foliations by Töben, Goertsches-Nozawa-Töben or
- the localization formula for K-contact manifolds due to Casselmann-Fisher.

Example

Theorem (Goertsches-N.-Töben)

Let $\mathcal{C}^*$ be the momentum polytope $\mathcal{C}^*$ of X. For each 1-dim T-orbit L, let $v_1^L, \dots, v_{n-1}^L$ be normal vectors of $\mathcal{C}^*$ such that $\Phi(L)(v_i^L) = 0$. Assume that the vectors $v_1^L, \dots, v_{n-1}^L$ are ordered so that $\det(b, v_1^L, \dots, v_{n-1}^L) > 0$. Then we have

$$\mathrm{vol}(M) = \frac{2\pi^n}{(n-1)!} \sum_L \frac{1}{\det(b, v_1^L, \dots, v_{n-1}^L)} \cdot \frac{\det(v, v_1^L, \dots, v_{n-1}^L)^{n-1}}{\prod_{i=1}^{n-1} \det(b, v_1^L, \dots, v_{i-1}^L, v, v_{i+1}^L, \cdots, v_{n-1}^L)},$$

where the right hand side is independent of $v \in \mathfrak{t}$.

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Thank you for your attention !