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Inverse source problem for Klein–Gordon equation in de Sitter space-time

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Abstract

Let $n \in \mathbb{N}$. De Sitter space-time is a solution of the vacuum Einstein equation with a positive cosmological constant in $1+n$ dimensional Euclidean space $(\mathbb{R}^{1+n}, g)$, where g is defined by

$$g = -dt \otimes dt + e^{2Ht} \sum_{j=1}^{n} dx^j \otimes dx^j$$

for a positive constant $H > 0$ and $(t, x^1, \dots, x^n)$ which indicates the Cartesian coordinate system. We consider Klein–Gordon equation in de Sitter space-time with a source term

$$\left(\Box_g + m^2\right) u = S\left(\int_0^t e^{-Hs} ds\right) f(x) \text{ in } (0,T) \times \mathbb{R}^n, \tag{1}$$

where $m, T > 0$ are constants, S, f are smooth functions and $\Box_g$ indicates d'Alembertian with respect to g. A lot of forward problems of the Klein–Gordon equation are studied. (e.g., [3], [5], [6])

In this talk, we consider inverse source problem for the Klein–Gordon equation (1) and prove a uniqueness theorem to determine a time-independent source term f up to a neighborhood of an open subset $\Omega \subset \mathbb{R}^n$, where a data of solution $u \mid_{[0,T) \times \Omega}$ is given when the mass term m in (1) has a particular value. Inverse source problems for equations having time-dependent coefficients are studied by [2]. Although they prove the local Hölder stability by Bukhgeim–Klibanov method [1], the method is no longer useful to our problem since a choice of a weight function is not given explicitly. We then see a new method based on the Duhamel's principle and theory of distributions with compact supports to deal with inverse problems for such equations having time-dependent coefficients.

References

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