A nonlinear integro-differential equation of earthquake faulting

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Dynamic rupture and slip between two rock masses during an earthquake have been modeled under a framework of an integral equation that represents traction force balance on a fault plane. Assuming that the earth’s crust is an infinite homogeneous elastic body with the density and rigidity of unity and its displacement \( u \) shows anti-plane deformation (e.g., \( u^T = (0, 0, u_3(x_1, x_2, t)) \)) for simplicity, the displacement is represented by the double layer potential [1]

\[
u_3(x_1, x_2, t) = \int_0^t \int_{\mathbb{R}} D(\xi, \tau) \partial_{x_2} G(x_1 - \xi, x_2, t - \tau) d\xi d\tau, \tag{1}\]

where \( D(x_1, t) := \lim_{\epsilon \to 0} [u_3(x_1, x_2, t)]_{x_2=+\epsilon}^{x_2=-\epsilon} \) is the slip along the fault (= \( x_1 \)-axis), and \( G(x_1, x_2, t) \) is the fundamental solution of the 2-D wave equation \( \partial_t^2 G - \Delta G = \delta(x_1, x_2, t) \) with zero initial condition \( (G = \partial_t G = 0 \text{ for } t \leq 0) \).

The shear stress \( \lim_{x_2 \to 0} \partial_{x_2} u_3 \) must balance with a change of frictional force on the fault. Many rock experiments have shown that the friction is a nonlinear, monotonically decreasing function of \( D \) and/or \( V := \partial_t D \); we may write \( \sigma = F(D, V) \) [2]. Simultaneously, an empirical law based on seismic observations allows us to assume that \( D \) and \( V \) are a homogeneous function of degree one and zero, respectively: i.e., \( D(x_1, t) = tD(x), \) and \( V(x_1, t) = V(X), \) where \( X := x_1/(vt), \) and \( v \) is the rupture velocity (i.e., extension speed of the support of \( D(x_1) \)) along the fault. Given the above properties, we derive the following nonlinear integro-differential equation

\[
\frac{X}{\sqrt{1 - (vX)^2}} \frac{d}{dX} F(D, V) = \frac{d}{dX} \int_{-1}^{+1} \frac{V(\xi)}{\xi - X} \frac{d\xi}{\pi} \tag{2}\]

from eq.(1) and exemplify approximate solutions of \( V \) for some simplified cases.

References
