A nonlinear integro-differential equation of earthquake faulting

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Dynamic rupture and slip between two rock masses during an earthquake have been modeled under a framework of an integral equation that represents traction force balance on a fault plane. Assuming that the earth's crust is an infinite homogeneous elastic body with the density and rigidity of unity and its displacement $\boldsymbol{u}$ shows anti-plane deformation (e.g., $\boldsymbol{u}^T = (0, 0, u_3(x_1, x_2, t))$) for simplicity, the displacement is represented by the double layer potential [1]

$$u_3(x_1, x_2, t) = \int_0^t \int_{\mathbb{R}} D(\xi, \tau) \partial_{x_2} G(x_1 - \xi, x_2, t - \tau) d\xi d\tau, \tag{1}$$

where $D(x_1, t) := \lim_{\epsilon \to 0} [u_3(x_1, x_2, t)]_{x_2=-\epsilon}^{x_2=+\epsilon}$ is the slip along the fault (= x_1-axis), and $G(x_1, x_2, t)$ is the fundamental solution of the 2-D wave equation $\partial_t^2 G - \Delta G = \delta(x_1, x_2, t)$ with zero initial condition ($G = \partial_t G = 0$ for $t \leq 0$).

The shear stress $\lim_{x_2 \to 0} \partial_{x_2} u_3$ must balance with a change of frictional force on the fault. Many rock experiments have shown that the friction is a nonlinear, monotonically decreasing function of D and/or $V := \partial_t D$; we may write $\sigma = F(D, V)$ [2]. Simultaneously, an empirical law based on seismic observations allows us to assume that D and V are a homogeneous function of degree one and zero, respectively: i.e., $D(x_1, t) = tD(X)$, and $V(x_1, t) = V(X)$, where $X := x_1/(vt)$, and v is the rupture velocity (i.e., extension speed of the support of $D(x_1)$) along the fault. Given the above properties, we derive the following nonlinear integro-differential equation

$$\frac{X}{\sqrt{1 - (vX)^2}} \frac{d}{dX} F(D, V) = \frac{d}{dX} \int_{-1}^{+1} \frac{V(\xi)}{\xi - X} \frac{d\xi}{\pi} \tag{2}$$

from eq.(1) and exemplify approximate solutions of V for some simplified cases.

References

[1] Aki, K. & P.G. Richards (1980), Quantitative seismology, 1st edn., Freeman, New York.

[2] Ohnaka, M. & T. Yamashita (1989), A cohesive zone model for dynamic shear faulting based on experimentally inferred constitutive relation and strong motion source parameters, *J. geophys. Res.*, **94**(B4) 4089–4104.

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