The uniqueness problem of Rayleigh wave in Kelvin viscoelastic half-space and possible method to solve the problem

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Abstract: Waves that propagate in elastic solid can be divided into two main categories: body waves and surface waves. Two and only two types of body waves in an unbounded solid, namely longitudinal wave (P-wave) and shear waves (SV-wave and SH-wave), can be propagated independently. Rayleigh wave is one kind of surface waves which propagates along the free boundary and decays exponentially away from the boundary. Rayleigh wave is essentially the formation of interference on the surface of the medium of P-wave and SV-wave. Therefore, it was firstly introduced as a solution of the free boundary problem for an elastic half space by Lord Rayleigh (1885) [1], who summed it up as the simultaneous solution of the equations of P-wave and SV-wave. Yet to this day, the method of the characteristic equation of surface wave velocity he deduced is still the important way to seek for Rayleigh wave [2].

It is important to realize that since a solution (Rayleigh wave) can be found using Elastic theory and Helmholtz decomposition, it can also be seen as the superposition of two separate components: one P-wave and the other SV-wave. So, we think it is necessary to review in detail the mechanical process and mathematical method of deducing Rayleigh wave so far. In this report another main attention is a new and simplified characteristic equation about Rayleigh wave, which is the result recently deduced with special aim at future purposes in viscoelastic media.

There remains a dispute about the number of Rayleigh waves in viscoelastic medium until now. Rayleigh wave in viscoelastic media was early and deeply studied by Scholte [3], who proofed that Rayleigh wave also existed on the surface of half-space viscoelastic media, but he was not sure whether the Rayleigh wave he calculated was the only one valid. In 1960, from the mathematical aspect of the effective characteristic root, Bland [4] raised the question of the existence and uniqueness of Rayleigh wave in viscoelastic media "It has not yet been shown that for any viscoelastic material there is one and only one such root" [4, p.75]. In response to this question, after a series of studies based on linear viscoelastic models, Currie et al. [5,6,7] came to a conclusion in 1977 that there was more than one possible Rayleigh wave in the viscoelastic half-space surface. In most cases, there existed two while in some special material there would even be three possible Rayleigh waves [6]. What's more, they predicted that the velocity of viscoelastic surface wave would sometimes be higher than that of the body wave, and there would even be retrograde wave [5,6].

After that, Bland's question seemed to be solved. But, on one hand, there is no report about the practical examples that the velocity of surface wave will surpass the body

wave velocity nor the retrograde propagation phenomenon. On the other hand, in 2001 Romeo [8,9] promoted Nkemzi's elastic Rayleigh wave equation [10] to linear viscoelastic situation, thus put forward an opposite viewpoint. He took advantage of complex modulus to analyze the root number of the characteristic equation of viscoelastic half-space Rayleigh wave. Although his derivation process contained multiple variable substitutions, which appeared complicated and obscure, he deemed that there is only one complex root valid, while the other two are invalid. This implied that there is only one truly valid Rayleigh wave in viscoelastic half-space surface.

Obviously, Romeo's opinion denied the conclusion by Currie et al., but it hasn't been widely accepted yet. Except that Ivanov and Savova's [11] study supports Romeo's opinion only in the sense of wavelength fixed condition, studies in the common sense of frequency fixed condition [12,13] and more researchers' studies [14,15,16] are in favor of the conclusion of Currie et al. The possibility of the existence of a retrograde Rayleigh surface wave is even suggested [16], and there maybe third types of Rayleigh waves in the case of some special combinations of material parameters [16]. What is worth noting is that the study in 2014 by Chirita et al [16]. is carried out based on Kelvin viscoelastic model, which is simple but very commonly adopted in practical engineering. It is interesting but a little unbelievable that the process of derivation, settings of variable and multiple variable substitutions by Chirita et al. in the study [16] were similar to Romeo's method, but in the conclusion, Romeo's opinion is completely negated. The obscure of mathematical derivation process in Chirita et al.'s article brings some difficulty to understanding for us, but the authors' work arouses our interest and makes us want to find out the truth.

This report is to propose a brief way of handling this essential problem within half-space Kelvin viscoelastic medium. Starting from the dynamic equations of transverse wave and longitudinal wave based on Kelvin viscoelastic model, we derive the characteristic equation of Rayleigh surface wave in Kelvin viscoelastic half-space in a much simpler way and in terms of complex wavenumber. We then propose a relatively convenient and concise method which is able to directly analyze the validity of characteristic roots in a physical sense. Finally, we reach the conclusion that there is only one Rayleigh wave in Kelvin viscoelastic half-space surface, confirming Romeo's [8,9] conclusion under the assumption of Kelvin model. Meanwhile, we point out the error of Chirita et al [16] have made in handling the result, negating their viewpoint that there is not only one Rayleigh wave in Kelvin viscoelastic half-space surface.

Keywords: Rayleigh wave; viscoelastic; wave equation; characteristic equation; Kelvin model

References:

[1] Rayleigh, L. On Waves Propagated along the Plane Surface of an Elastic Solid. *Proceedings of the London Mathematical Society*, **17**, 4-11 (1885)

[2] Eric A. Ash, Edward G. Paige. Rayleigh-wave Theory and Application. *Proceedings of an International Symposium Organized by The Rank Prize Funds at The Royal Institution*, London, 15–17 July (1985)

[3] Scholte, J. G. On Rayleigh waves in visco-elastic media. *Physica*, **13**, 245-250 (1947)
[4] Bland, D. R. *The Theory of Linear Viscoelasticity*, Pergamon, Oxford (1960)
[5] Currie, P. K., Hayes, M. A., O'Leary, P. M. Viscoelastic Rayleigh waves. *Quarterly of Applied Mathematics*, **35**, 35-53 (1977)
[6] Currie, P. K., O'Leary, P. M. Viscoelastic Rayleigh waves II. *Quarterly of Applied Mathematics*, **36**, 445-454 (1978)
[7] Currie, P. K. Viscoelastic Surfaee waves on a Standard Linear Solid. *Quarterly of Applied Mathematics*, **37**, 332-336 (1979)
[8] Romeo, M. Rayleigh waves on a viscoelastic solid half-space. *Journal of the Acoustical Society of America*, **110**, 59-67 (2001)
[9] Romeo, M. Uniqueness of the solution to the secular equation for viscoelastic surface waves. *Appl. Math. Lett.*, **15**, 649-653 (2002)
[10] Nkemzi, D. A new formula for the velocity of Rayleigh waves. *Wave Motion*, **26**, 199-205 (1997)
[11] Ivanov, T. P., Savova, R. Viscoelastic surface waves of an assigned wavelength. *European Journal of Mechanics - A/Solids*, **24**, 305-310 (2005)
[12] Ivanov, T. P., Savova, R. Viscoelastic surface waves. *European Journal of Mechanics - A/Solids*, **12**, 667-677 (1993)
[13] Ivanov, T. P., Savova, R. Motion of the particles due to viscoelastic surface waves of an assigned frequency. *Mathematics and Mechanics of Solids*, **19**, 725-731 (2014)
[14] Carcione, J. M., José, M. Rayleigh waves in isotropic viscoelastic media. *Geophysical Journal International*, **108**, 453-464 (1992)
[15] Savova, R., Ivanov, T. P. On viscous effects in surface wave propagation. *J. Theor. Appl. Mech.*, **35**, 25-34 (2007)
[16] Chirita, S., Ciarletta, M., Tibullo V. Rayleigh Surface Waves on a Kelvin-Voigt Viscoelastic Half-Space. *Journal of Elasticity*, **115**, 61-76 (2014)