On unilateral contact problems with friction for an elastic body with cracks

Hiromichi Itou

Department of Mathematics, Tokyo University of Science

We discuss about frictional contact problems in two dimensional elasticity, which are arising from a wide variety of phenomena in mechanical engineering and concerning with some inverse problems and control problems. Contact conditions for cracks are so-called non-penetration conditions defined as unilateral conditions on the displacements of bodies to exclude nonphysical phenomenon such as mutual penetration of crack faces, see [1] for the details. In this talk, we introduce mathematical results obtained in [5] and [6]. First, in [5] Coulomb friction in the static case is considered and the existence of the solution is shown by using penalization method. Moreover, we derive asymptotic expansions of the solution near the crack tip. Second, an interfacial crack of two elastic bodies is considered in [6]. On the crack the non-penetration condition and Tresca friction condition which is an approximation of Coulomb friction (e.g. [2]) are imposed. On the other part of the interface, both of adhesion forces and friction force are taken into account. Then we derive a formula for the derivative of the energy functional with respect to the crack length, which can be represented as a path-independent integral (J -integral) and also some numerical results are introduced. Finally, we mention about dynamic unilateral contact problems. In my knowledge, there are a few mathematical results dealing with the existence of a solution to such kinds of problems (cf. [3, 4, 7]). Then, we overview the related known results and clarify the main difficulty.

Acknowledgments

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