A NUMERICAL METHOD FOR INVERSE TRANSPORT PROBLEMS

MANABU MACHIDA

Inverse problems of estimating coefficients of the radiative transport equation, which is the linear Boltzmann equation, will be considered. In particular in optical tomography, the spatial distributions of coefficients are reconstructed from boundary measurements. Near-infrared spectroscopy (NIRS) includes an application of inverse transport problems. NIRS obtains information on biological tissue by studying the reflected light when the sample is illuminated by a near-infrared light source. The near-infrared light in biological tissue obeys the radiative transport equation. Although the diffusion equation has been used as an approximation of the radiative transport equation in most studies in NIRS, in this talk we will consider the radiative transport equation, which is given by

$$\begin{cases} (\theta\cdot\nabla+\mu_a+\mu_s)\,u(x,\theta)-\mu_s\displaystyle\int_{\mathbb{S}^2}p(\theta,\theta')u(x,\theta')\,d\theta'=0, & (x,\theta)\in\Omega\times\mathbb{S}^2, \\ \qquad\qquad\qquad\qquad\qquad\qquad u(x,\theta)=g(x,\theta), & (x,\theta)\in\Gamma_-, \end{cases}$$

where $\Omega\subset\mathbb{R}^3$ and $\Gamma_\pm=\{(x,\theta)\in\partial\Omega\times\mathbb{S}^2;\ \pm\theta\cdot\nu(x)>0\}$ with the outer unit normal $\nu(x)$. The incident beam is denoted by $g(x,\theta)$. We note that the above radiative transport equation is independent of time and describes stationary transport. Even when time-dependent light pulses are used for measurements, the problem reduces to an inverse problem for the time-independent radiative transport equation after the data is integrated over time (such as the Fourier transform, Laplace transform, or Mellin transform). The goal of NIRS, which includes optical tomography, is to determine the absorption coefficient $\mu_a(x)$, the scattering coefficient $\mu_s(x)$, or both through inverse transport problems. In this talk, we assume that the scattering phase function $p(\theta,\theta')$ does not vary in space and is given.

Institute for Medical Photonics Research, Hamamatsu University School of Medicine, Hamamatsu 431-3192, Japan

E-mail address: machida@hama-med.ac.jp.