

Speaker: Xiang Xu

Title: Inversion Trace Formulas for a Sturm-Liouville Operator

Abstract: This talk revisits the classical problem "Can we hear the density of a string?", which can be formulated as an inverse spectral problem for a Sturm-Liouville operator. Instead of inverting the map from spectral data to density directly, we propose a novel method to reconstruct the density, based on inverting a sequence of trace formulas which bridge the spectral data and the density clearly in terms of a series of nonlinear Fredholm integral equations. The main difficulties of the new method lie in two aspects. One is to compute effectively multi-dimensional integrals in trace formulas. The other is to propose an appropriate measure to distinguish the trace differences with respect to different operators (densities) stably. For the first difficulty, Fourier expansion is utilized both for the kernel function and the unknown density to reduce computational efforts tremendously. For the second difficulty, shifted Chebyshev polynomials are adopted to appropriately measure the differences. Numerical experiments are presented to verify the validity and effectiveness of the proposed numerical algorithm. The impact of different parameters involved in the algorithm is also discussed.