Almost complex torus manifolds - graphs, Hirzebruch genera, and problem of Petrie type

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Let a k-dimensional torus T^k act on a $2n$-dimensional compact connected almost complex manifold M with isolated fixed points. We show that there exists a (directed labeled) multigraph that encodes weights at the fixed points of M. If in addition $k = n$, i.e., M is an almost complex torus manifold, the multigraph is a graph; it has no multiple edges. We show that the Hirzebruch χ_y-genus $\chi_y(M) = \sum_{i=0}^n a_i(M) \cdot (-y)^i$ of an almost complex torus manifold M satisfies $a_i(M) > 0$ for $0 \leq i \leq n$.

Petrie's conjecture asserts that if a homotopy $\mathbb{CP}^n$ admits a non-trivial circle action, its Pontryagin class agrees with that of $\mathbb{CP}^n$. Petrie proved this conjecture if instead it admits a T^n-action. Using the above, we prove that if a $2n$-dimensional almost complex torus manifold M only shares the Euler number with the complex projective space $\mathbb{CP}^n$, an associated graph agrees with that of a linear T^n-action on $\mathbb{CP}^n$; consequently M has the same weights at the fixed points, Chern numbers, equivariant cobordism class, Hirzebruch χ_y-genus, Todd genus, and signature as $\mathbb{CP}^n$. If furthermore M is equivariantly formal, the equivariant cohomology and the Chern classes of M and $\mathbb{CP}^n$ also agree.