On an overdetermined problem for composite materials

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In his celebrated paper [Se], Serrin showed that the overdetermined problem

$$-\Delta u = 1 \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega, \quad \partial_n u = \textit{constant} \text{ on } \partial\Omega$$

is solvable if and only if the domain Ω is a ball. In this talk we will explore how this result changes when the Laplace operator in the overdetermined problem above is replaced by the operator in divergence form $\Delta_\sigma = \operatorname{div}(\sigma\nabla\cdot)$, where σ denotes the piecewise constant function that takes the value σ_c on a given subdomain $D \subset\subset \Omega$ and 1 otherwise (here σ_c is a given positive constant). We will show that this new overdetermined problem admits infinitely many solutions that do not enjoy radial symmetry. We will study how the shape of D affects the geometry of these "non-trivial" solutions, and explain how they relate to the critical points of some shape functional.

In particular, we will focus our attention on the following two approaches.

- Mathematically construct some of these non-trivial solutions as perturbations of trivial (i.e. radially symmetric) ones by making use of the implicit function theorem ([CY1]) and the Crandall–Rabinowitz theorem ([CY2]).
- Compute such solutions numerically by means of a steepest descent algorithm based on a Kohn–Vogelius type functional ([CY1]).

If time permits, we will also discuss some open questions and some work in progress concerning more recent developments of this problem.

This is joint work with Toshiaki Yachimura (Kyoto University).

References

[Se] J. Serrin, Arch. Ration. Mech. Anal. 43 (1971), 304–318.

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