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### 1. Introduction

With the highest incidence occurring in women, breast cancer causes about 520,000 deaths worldwide [1] and about 14,000 deaths in Japan alone [2]. If discovered early, the 5-year survival rate is 93% [3]. Throughout the history of development of breast cancer screening technology, there is no doubt that Xray mammography has survived as the most superior breast-cancer-screening technique in comparison to other techniques, including ultrasound, nuclear magnetic resonance imaging, and positron emission tomography. Unfortunately, despite advances in the field, X-ray mammography is still deficient when it comes to early and accurate detection of tumours, especially in cases where breast tissue densities are higher than average. This is because when collagen fibres are compact and highly concentrated, a condition known as "dense breasts," X-rays cannot produce the contrast of the breast tumor due to both the large X-ray absorption and its large fluctuation of the collagen density, thus smaller cancer tumours are not easily detected. Since dense breasts are found in about 79% of women under 50 in Asia (61% in Caucasians, 57% in Black, and 51% in Hispanic) [4], a new breast-cancer-imaging technique is desperately needed.

## 2. Electromagnetic Properties of Breast Cancer

Breasts are mostly fat tissues by volume; the aforementioned collagen fibres and mammary glands, which serve to generate and transport breast milk, are found in these fat tissues. Unfortunately, cancer tumours can infiltrate the abovementioned mammary glands, where they have easy access to the surrounding tissues and are well supplied with blood capillaries. Interestingly, when this is viewed in light of the theory of electromagnetic properties, most breasts largely comprise insulating macromolecules with low relative permittivity, whereas breast-cancer tumours have a high relative permittivity. If we consider such properties, it becomes clear that the microwave range, with relatively long wavelengths that do not absorb well at the molecular level, is ideal for visualizing breast cancer tumours.

However, microwave mammography cannot be achieved if one important problem is not first addressed: the issue of inverse wave scattering. The challenge of visualizing breast cancer tumours when breasts are irradiated with microwave, referred to as the "scatterer," lies in the fact the microwave are scattered in various directions by breast cancer tumours with high relative permittivity. In other words, if we could develop a mathematical theory and algorithm that visualizes the three-dimensional structures of a scatterer based on multi-static (multi-static) scattering data obtained on a curved surface such as the surface of a breast, the use of microwave mammography as a tool in early detection could be realized [5]. In our research, we have achieved to establish the world's first multi-static inverse scattering theory against the unresolved issue.

## 3. Four-dimensional Partial Difference Equation of Scattering Field

In this solution, it is assumed that the visualization target is a cone with a linear generatrix (Fig. 1); thus, antenna elements are linearly arranged. Let us consider the tangent plane of a cone, including its array antennas. On this tangent plane, we define the x axis on the reference plane, the axis of the antennas as the y axis, and the z axis is seen as being perpendicular to these two axes. Thus, the x axis rotates on the reference plane by  $\theta$  degrees around the Z axis from the X axis. Assuming that the hypothetical tangent plane rotates and the scattering data obtained for all  $\theta$ , we present the theory for reconstructing three-dimensional structures within a cone from multi-static time-series data on the cone's surface

If we define the multi-static coordinate  $y_1$ ,  $y_2$  on the y axis, a reconstruction theory can be established on the fact that data are only obtained at one point  $(x=0,y_1,y_2)$  for the x coordinate along the tangent plane.

Below, we describe a method for obtaining the relevant differential operator. Let us think of a situation in which one-dimensional array antenna (Fig. 2) shifts to the x axis direction along the x-y plane. As shown in Fig. 2, let us consider a situation wherein a microwave emitted from the point  $P_1(x,y_1,z)$  is reflected at the point  $P(\xi,\eta,\zeta)$  and received at the point  $P_2(x,y_2,z)$ . When Point P moves throughout Range

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Fig. 1 Linear antenna array and a cone



Fig. 2 One-dimensional array antenna

D, the signal received at  $P_2$  can be expressed as follows:

$$\varphi(x, y_1, y_2, z) = \iint_D \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \varepsilon(\xi, \eta, \zeta) d\xi d\eta d\zeta$$
  

$$\rho_1 = \sqrt{(x-\xi)^2 + (y_1 - \eta)^2 + (z-\zeta)^2}$$
(1)  

$$\rho_2 = \sqrt{(x-\xi)^2 + (y_2 - \eta)^2 + (z-\zeta)^2}$$

where k is the wave number of the microwave, and the factor of time is assumed to be relative to  $\exp(-i\omega t)$ . Let the kernel function in the integration term of the above Equation  $\phi$ , then:

$$\phi = \frac{e^{ik\rho_1}}{\rho_1} \frac{e^{ik\rho_2}}{\rho_2} \tag{2}$$

Thus, a partial differential equation is obtained in

which this equation or the differentiation and integration of this equation with respect to  $\xi, \eta, \zeta$  gives an asymptotic solution. To that end, higher order terms for  $1/\rho$  created by the differentiation should be ignored in the calculation; we then define the abbreviation for differentiation as

$$\frac{\partial}{\partial t} \to \partial_t, \frac{\partial}{\partial x} \to \partial_x, \frac{\partial}{\partial y_1} \to \partial_{y_1}, \frac{\partial}{\partial y_2} \to \partial_{y_2}, \frac{\partial}{\partial z} \to \partial_z \quad (3)$$

Then, we obtain the following equation about the kernel function  $\phi$ :

$$\begin{bmatrix} \frac{1}{4} \{ \Delta_4 - 2(ik)^2 \}^2 - \partial_{y_1}^2 \partial_{y_2}^2 + \\ (ik)^2 (\partial_{y_1}^2 + \partial_{y_2}^2) - (ik)^4 \end{bmatrix} \phi = 0$$
 (4)

Though this equation is derived under the assumption of steady state, it can be easily expanded to include an unsteady state by substituting a variable as seen below:

$$-ik \to \frac{1}{c}\partial_t$$
 (5)

Where c is the speed of light. Ultimately, we obtain the following equation:

$$\{\Delta_{4}{}^{2} - \frac{4}{c^{2}}(\partial_{t}{}^{2}\partial_{x}{}^{2} + \partial_{t}{}^{2}\partial_{z}{}^{2}) - 4\partial_{y_{1}}^{2}\partial_{y_{2}}{}^{2}\}\phi = 0$$

$$\Delta_{4} = \partial_{x}{}^{2} + \partial_{y_{1}}{}^{2} + \partial_{y_{2}}{}^{2} + \partial_{z}{}^{2}$$
(6)

By applying the derivative to the integral kernel,  $\varphi$  satisfies the above-described partial differential equation. This equation is a four-dimensional psuedo-wave equation consisting of five variables  $(t, x, y_1, y_2, z)$ .

# 4. Analytical Solution and Three-dimensional Structure Image of Scatter

Let us now solve this equation using Fourier transform. First, we perform multiple Fourier transform of  $\varphi$  with respect to  $t, x, y_1, y_2$ 

$$\tilde{\varphi}(k_x,k_{y_1},k_{y_2},z,\omega) =$$

$$\int_{-\infty}^{\infty} e^{i\omega t} dt \int_{-\infty}^{\infty} e^{ik_{y_1}y_1} dy_1 \int_{-\infty}^{\infty} e^{ik_{y_2}y_2} dy_2 \int_{-\infty}^{\infty} e^{ik_x} \varphi(x, y_1, y_2, z, t) dx$$
(7)

If we use  $D_z$  for the differentiation with respect to z, we obtain the following formula:

$$\{(D_{z}^{2}-k_{x}^{2}-k_{y_{1}}^{2}-k_{y_{2}}^{2})^{2}+4k^{2}(D_{z}^{2}-k_{x}^{2})-4k_{y_{1}}^{2}k_{y_{2}}^{2}\}\tilde{\varphi}=0$$
(8)

where  $\omega = c \ k$  is used, the four fundamental solutions for this equation are

$$E_{1} = e^{i\{\sqrt{(\sqrt{k^{2}-k_{y_{1}}^{2}}+\sqrt{k^{2}-k_{y_{2}}^{2}})^{2}-k_{x}^{2}}z}}$$

$$E_{2} = e^{-i\{\sqrt{(\sqrt{k^{2}-k_{y_{1}}^{2}}+\sqrt{k^{2}-k_{y_{2}}^{2}})^{2}-k_{x}^{2}}z}}$$

$$E_{3} = e^{i\{\sqrt{(\sqrt{k^{2}-k_{y_{1}}^{2}}-\sqrt{k^{2}-k_{y_{2}}^{2}})^{2}-k_{x}^{2}}z}}$$

$$E_{4} = e^{-i\{\sqrt{(\sqrt{k^{2}-k_{y_{1}}^{2}}-\sqrt{k^{2}-k_{y_{2}}^{2}})^{2}-k_{x}^{2}}z}}$$
(9)

Considering that the temporal factor is  $e^{-i\omega t}$ , phase is added by the microwave path and that the microwave reflected by an object bounces back to the measurement surface, we obtain  $E_1$  as the only meaningful solution. Therefore,

$$\tilde{\varphi}(k_x, k_{y_1}, k_{y_2}, z, k) =$$

$$a(k_x, k_{y_1}, k_{y_2}, k)e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2}\}z}$$
(10)

By applying Fourier transform to the above equation, we obtain the solution  $\varphi$  for the wave Equation (6):

$$\varphi(x, y_1, y_2, z, k) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y_2)} a(k_x, k_{y_1}, k_{y_2}, k)$$
$$e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}}^2 + \sqrt{k^2 - k_{y_2}}^2)^2 - k_x^2}\}^z} dk_x dk_{y_1} dk_{y_2}}$$
(11)

Let us now consider that measured data only exists for x = 0; thus, the following equation stands:

$$\varphi(x, y_1, y_2, 0, k) = \varphi_R(y_1, y_2, k)\delta(x) \tag{12}$$

If we apply this to the above equation with z = 0, we then obtain

$$\varphi_{R}(y_{1}, y_{2}, k)\delta(x) = \frac{1}{(2\pi)^{3}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-i(k_{x}x + k_{y_{1}}y_{1} + k_{y_{2}}y_{2})} a(k_{x}, k_{y_{1}}, k_{y_{2}}, k)dk_{x}dk_{y_{1}}dk_{y_{2}}$$
(13)

Applying Fourier transform by  $(x, y_1, y_2)$  on both sides of the equation, we obtain the following:

$$a(k_x, k_{y_1}, k_{y_2}, k) = \tilde{\varphi}_R(k_{y_1}, k_{y_2}, k)$$
(14)

Thus, we obtain the solution  $\varphi(x, y_1, y_2, z, k)$  of the partial differential equation. We can then take Trace with respect to  $y_1, y_2$  of the function  $\varphi(x, y_1, y_2, z, k)$ , specifically, if the visual function is defined as  $y_1 \rightarrow y, y_2 \rightarrow y$ , where the term k, z is fixed:

$$\begin{split} \varphi(x,y,y,z,k) &= \lim_{y_1 \to y} [\varphi(x,y_1,y,z,k)] \\ &= \lim_{y_1 \to y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \\ e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y)} a(k_x,k_{y_1},k_{y_2},k) \\ &e^{iz \left\{ \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2} \right\}} dk_x dk_{y_1} dk_{y_2}] \end{split}$$

$$= \lim_{y_1 \to y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx \right]$$

$$e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y)} \tilde{\varphi}_R(k_{y_1}, k_{y_2}, k)$$

$$e^{iz} \left\{ \sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2} \right\} dk_x dk_{y_1} dk_{y_2} \right]$$

$$= \lim_{y_1 \to y} \left[ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dk_{y_1} dk_{y_2} dk_{y_2} dk_{y_1} dk_{y_2} dk_{$$

We can now convert the variables to integrate with respect to k:

$$k_{z} = \sqrt{\left(\sqrt{k^{2} - k_{y_{1}}^{2}} + \sqrt{k^{2} - k_{y_{2}}^{2}}\right)^{2} - k_{x}^{2}}$$

$$k = \frac{1}{2}\sqrt{k_{x}^{2} + k_{z}^{2} + \frac{\left(k_{y_{1}}^{2} - k_{y_{2}}^{2}\right)^{2}}{k_{x}^{2} + k_{z}^{2}}} + 2\left(k_{y_{1}}^{2} + k_{y_{2}}^{2}\right)$$

$$\frac{dk}{dk_{z}} = \frac{k_{z}\sqrt{k^{2} - k_{y_{1}}^{2}}\sqrt{k^{2} - k_{y_{2}}^{2}}}{k\left(k_{x}^{2} + k_{z}^{2}\right)}$$
(16)

Next, we perform Fourier transform on Equation (15) with respect to k, and apply t = 0. Then, we obtain the visualization function  $\rho(r,\theta)$  in the localized coordinate system at the angle  $\theta$  in Equation (17). We describe  $\theta$  dependency of  $\tilde{\varphi}_R(k_{y_1},k_{y_2},k)$  positively as  $\tilde{\varphi}_R(k_{y_1},k_{y_2},k,\theta)$ 

$$\rho(x,y,z,\theta) = \int_{-\infty}^{\infty} \varphi(x,y,y,z,k) dk 
= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\varphi(x,y_1,y,z,k)] dk 
= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_{y_1} dk_{y_2} dk 
e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y)} a(k_x, k_{y_1}, k_{y_2}, k) 
e^{i\{\sqrt{(\sqrt{k^2 - k_{y_1}^2} + \sqrt{k^2 - k_{y_2}^2})^2 - k_x^2\}z}} dk_x dk_{y_1} dk_{y_2}] dk 
= \int_{-\infty}^{\infty} \lim_{y_1 \to y} [\frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk_x dk_{y_1} dk_{y_2} dk_z] 
e^{-i(k_x x + k_{y_1} y_1 + k_{y_2} y)} e^{ik_z z} \tilde{\varphi}_R(k_{y_1}, k_{y_2}, k, \theta) 
\left(\frac{dk}{dk_z}\right) dk_x dk_{y_1} dk_{y_2} dk_z]$$
(17)

By integrating results for each  $\theta$ , we obtain images of three-dimensional structures.

# 5. Conversion from Tangent Space to the Whole Coordinate

We then convert the results calculated in the tangent space to the whole coordinate (X, Y, Z). If we consider the projection of y axis on the (X, Y) plane as y', the following equations are established:

$$y = y' \cos \alpha + Z \sin \alpha$$

$$z = -y' \sin \alpha + Z \cos \alpha$$
(18)

Where  $\alpha$  is the angle between the coordinate axis z of the tangent plane and the Z axis of the reference plane. The equations converting from (x,y') to (X, Y) are

$$\begin{aligned} x &= X\cos\theta + Y\sin\theta \\ y' &= -X\sin\theta + Y\cos\theta \end{aligned}$$
 (19)

Summarizing the above gives

$$x = X\cos\theta + Y\sin\theta$$
$$y = -X\cos\alpha\sin\theta + Y\cos\alpha\cos\theta + Z\sin\alpha$$
(20)

 $z = X \sin \alpha \sin \theta - Y \sin \alpha \cos \theta + Z \cos \alpha$ 

We apply these conversion formulas to the following equation:

$$\rho(x,y,z,\theta) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dk_x dk_{y_1} dk_{y_2} dk_z dk_{y_1} dk_{y_2} dk_z$$

$$\left(\frac{dk}{dk_z}\right) dk_x dk_{y_1} dk_{y_2} dk_z$$
(21)

We introduce the following variables:

$$k_{ye} = k_{y_1} + k_{y_2}$$

$$k_{yo} = k_{y_1} - k_{y_2}$$
(22)

The above equation can be written as

We perform the following variable conversions in the spectral range:

$$\xi = -k_x \cos\theta + (k_{ye} \cos\alpha + k_z \sin\alpha) \sin\theta$$
  

$$\eta = -k_x \sin\theta - (k_{ye} \cos\alpha + k_z \sin\alpha) \cos\theta \qquad (24)$$
  

$$\varsigma = -k_{ye} \sin\alpha + k_z \cos\alpha$$

This inverse transformation gives the following equations:

$$k_x = -\xi \cos\theta - \eta \sin\theta$$
$$k_{ye} = (\xi \sin\theta - \eta \cos\theta) \cos\alpha - \varsigma \sin\alpha$$
(25)

 $k_z = (\xi \sin \theta - \eta \cos \theta) \sin \alpha + \varsigma \cos \alpha$ 

Then the reconstruction function for angle  $\theta$  becomes

Since  $k_x, k_{y1}, k_{y2}$ , and kz are functions with respect to  $\xi, \eta, \varsigma$ , and  $k_{yo}$  as in Equations (22) and (25), these can be converted by performing Fourier transform on data localized in the coordinate of angle  $\theta$  on the whole coordinate. Finally, the reconstruction image is obtained by integrating with respect to the angle  $\theta$ :

$$P(X,Y,Z) = \int_{0}^{2\pi} \rho(x,y,z,\theta) d\theta$$
$$= \frac{1}{2(2\pi)^{3}} \lim_{y_{o} \to 0} \int_{0}^{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$$
$$e^{i(\xi X + \eta Y + k_{y_{o}}y_{o} + \varsigma Z)} \tilde{\varphi}_{R}(k_{y_{1}},k_{y_{2}},k,\theta)$$
$$\left(\frac{dk}{dk_{z}}\right) d\xi d\eta dk_{y_{o}} d\varsigma d\theta \tag{27}$$

In this paper, we describe the multi-static inverse scattering theory that we have developed successfully as the core application for microwave mammography [5–7]. In clinical trials using a 1–14 GHz ultra-wideband microwave mammography system [6,7] with over 350 breast cancer patients and healthy individuals, according to the physics, high detection sensitivity was shown in dense breasts. Fig. 3 shows the demonstration our microwave mammography image. In the fu-



Fig. 3 Microwave mammography image[5]

ture, we hope that this system will become widespread, thus helping many breast-cancer patients.

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