Mathematics of Image Reconstruction in Sparse-view CT and Interior CT

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Abstract

Since 2000, it has been widely recognized that radiation dose in CT examinations increases cancer risk. To overcome this drawback, new designs of CT scanners called sparse-view CT and interior CT have been actively investigated in CT community. The sparse-view CT refers to CT in which the number of projection data is reduced to decrease patient dose as well as to accelerate data acquisition. The interior CT refers to CT in which x-rays are radiated only to a small region of interest (ROI) to decrease patient dose. A key in these scanners is how to reconstruct images with sufficient quality from the limited projection data. This talk mainly consists of three parts.

The first part is concerned with image reconstruction for the sparse-view CT using Compressed Sensing (CS). CS is a promising technique appeared around 2005, which is able to reconstruct high-quality images even from the limited number of projection data. The explanation will be constructed as follows. First, we explain basic knowledge on this subject, which includes the principle of sparse-view CT, as well as the principle of CS image reconstruction. Second, we introduce our two CS image reconstruction methods developed for the sparse-view CT. The first method is based on the Total Variation (TV) regularization. The second method is called the second-generation CS, in which the regularization term is designed by using Non-Local-Mean denoising filter. We also show application examples to medical x-ray CT and x-ray phase CT.

The second part is concerned with image reconstruction for the interior CT. For a long time up to 2007, it had been believed that exact image reconstruction in the interior CT is impossible, because the Radon transform operator corresponding to the interior CT possesses a complicated null space. Since 2007, however, several exact solution methods have been discovered in the CT community. To guarantee the solution uniqueness, some of them use very small prior knowledge on the object, and others use very small additional measurement of projection data. We introduce these research activities including ours toward the exact interior tomography. We also show application examples to medical x-ray CT and x-ray phase CT.

The third part is concerned with mathematics of Region of Interest (ROI) reconstruction. The exact solution of the interior CT reconstruction was discovered by the two papers in medical physics field 12 years ago. It is known that the image

reconstruction in the interior CT is a special case of a more general image reconstruction problem called the ROI reconstruction. We would like to introduce the history including some more rigorous mathematics on how the ROI reconstruction problem was solved leading to the exact solution of the interior CT.