Pointwise error estimation and high-precision resistivity measurement with four-probe method

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Abstract

To measure the resistivity of semiconducor materials is one of the fundamental problems in the semiconductor industry. In this research, we study the mathematical models of resistivity measurement method and propose new computation algorithms along quantitative error estimation to calculate several fundamental quantities required by the resistivity measurement. Particularly, as a general mathematical theory on the pointwise error estimation for the finite element approximate solution to partial differential equation, we have developed a new theory to provide an optimal and explicit error estimation.

1 Introduction

The four-point probe method is widely used in the resistivity measurement. The principle of the four-point probe method is illustrated as in Figure 1, where the equidistant four probes A, B, C and D are aligned on the surface of a semiconductor object and a constant current I_{AD} is imposed on (A, D) pair and the potential difference V_{BC} between (B, C) pair is measured.

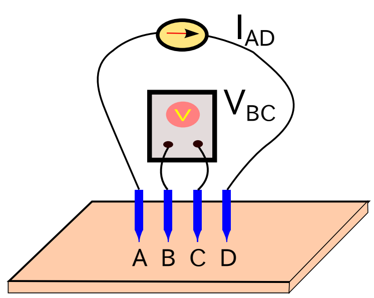

Figure 1: Four-point probe method for semiconductor resistivity measurement

Suppose the semiconductor object has a uniform distribution of resistivity ρ. The resistivity of the semiconductor object is expected to be calculated by using V_{BC} and I_{AD},

$$\rho = F_c \cdot \frac{V_{BC}}{I_{AD}},$$

where F_c is called by "correction factor". The factor F_c dependends on the geometric shape of the object, the position of the probes and the layout of probes A, B, C and D. The determination of F_c is reduced to solving the equation that describes the potential distribution.

Assume that the probes contact with the surface of material at points with zero contact area. Alghough such an assumption is not available in practical measurement, it works as a reasonable model to solve practical problems. The equation of potential Φ is given by

$$\Delta\Phi(x) = 2\rho\, I_{AD}\; (\delta(x-D) - \delta(x-A)) \;\text{ in } \Omega; \qquad \frac{\partial \Phi}{\partial n} = 0 \text{ on } \partial\Omega \tag{1}$$

where δ is the Dirac delta function in $\mathbf{R}^3$. By taking $\rho\, I_{AD} = 1$, the concrete value of F_c can be obtained as follows:

$$F_c = \frac{1}{\Phi(B) - \Phi(C)}\,.$$

For object of regular shapes, the equation is solved with explicit formula and the results are accepted by the JIS standard; see the discussion in , e.g., [1].

2 Problems of current resistivity measurement method

The model in (1) is developed under ideal conditions. In practical resistivity measurement, there are several factors that affect the precision of the measurement results.

- **Irregular shapes of the wafers** The wafers in general processing of semiconductor industry usually have a cut edge or notch to mark the orientation; see Figure 2. Since existing methods regard the wafer as a disk, the measurement error around the cut edge or notch is unignorable. To have a trustable measurement, one has to solve the equation (1) over an iiregular shape by numerical scheme, for example, the finite element method.

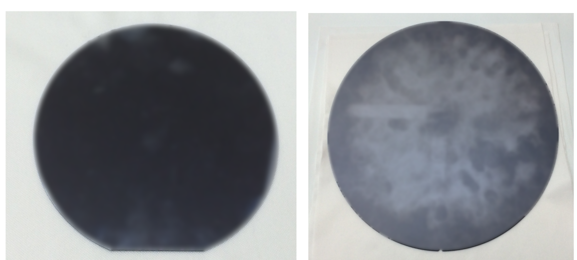

Figure 2: Wafers with cut edge (left) and notch (right).

- **The contact condtion of the probes** Although the end of the proble is very sharp, the contact area between the probe and the material is not ignorable (See Figure 3) and the affection of the non-zero contact area should be properly estimated for the purpose of high-precision mesaurement. To deal with the non-zero contact area problem, one has to set up new mathemtatical model upon the contact condition. For example, a reasonable assumption is that there is a equipotential on the contact area of each proble, which leads to the Dirichlet boundary condition of the governing equation.

- **Non-uniforma distribution of resistivity** Generally, the distribution of resistivity of semiconductor wafers is not uniform. Current method first measures the

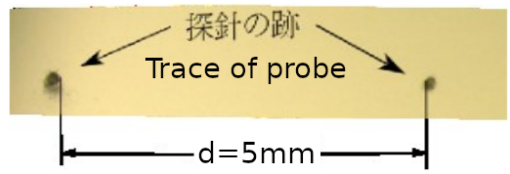

Figure 3: Trace of probes

resistivity at one point under the assumption of a uniform distribution of resistivity, then estimates the quality of wafers by analyzing the variation of measured values of resistivity. To give more reasonable model for the practical measurement, one has to consider the non-uniform distribution of resistivity. For this purpose, the optimiazation model will be needed to find the distribution of resistivity, along with the usage of more measurements results on the whole surface of the material.

3 Challenges to numerical analysis

By applying numerical schemes, one can easily obtain approximate solution to the model problem (1) or updated models with equipotential on the contact area. In this case, the error estimation for the numerical approximate solution is of great importance. Most classical numerical analysis only provides qualitical results such like convergence order, while a concrete value of the error is usually not available.

In our research, we developed new quantitative error estimation to provide explicit and sharp error estimation for the numerical results. In [3], Liu extended the idea of Kikuchi about the *a posteriori* error estimation [2] to develop a quantitative *a priori* error estimation for the boundary value problem of Poisson's equation. In [4], rather than a global error estimation of the numerical solutions, we focus on the local sub-domain around the probes and have developed the local error estimation thoery to provide explicit local error estimations under the energy norm.

Notice that the evaluaton of F_c requires the pointwise values of solution on the probe B and C. However, to have an explicit error estimation of the pointwise vaule of solution to partial differential equation is not an easy task. In our research, we consider the explicit estimation of ponitwise value of solutions by using the hypercircle method. Although the L^∞ norm estimation can produce uniform bound over the whole domain, the $O(h^2|log(h)|)$ convergence rate of L^∞ is an overestimated result for solution value at interior point of the domain.

In 1950s, T. Kato considered a kind of boundary value problem associated with a self-adjoint operator H of Hilbert spaces defined in the form $H = T^*T$, where T^* denotes the self-adjoint operator of T [5]. Later, H. Fujita applied the theory of Kato to develop a hypercircle-like method for error estimation and further applied this method to provide pointwise error estimation for the boundary value problems [6]. The method proposed in [6] only consider the model problem over a square domain along with the special base functions.

In our research, we studied the two fundamental papers [5,6], and investigated the possibility to apply Kato-Fujita's theory to develop pointwise error estimation for FEM solutions to boundary value problems. It is shown that, for linear conforming FEM, one can obtain explicit lower and upper bound for the solution value at a specified point inside the domain with $O(h^2)$ convergence rate.

In the presentation of this RIMS workshop, more details will be provided on the newly developed local error estimation and the point-wise error estimation.

Acknowledgement:
This reasearch is a joint work with NAPSON Corporation.

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