Reconstruction Problems in Algebraic Vision (abstract)

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In this introductory talk, I will present an algebro-geometric formulation of reconstruction problems in computer vision, emphasizing the aspect that these are inverse problems.

We consider the general problem of reconstructing information of objects in the world-space from the images of multiple cameras with unknown configuration. Here, the cameras are described by the pinhole camera model, the simplest mathematical model of perspective projection. It models a camera as a linear projection between projective spaces,

$$\varphi\colon \mathbb{P}^n \dashrightarrow \mathbb{P}^m \quad (n > m), \tag{1}$$

and the world-space as an open subset $\mathbb{R}^n$ of the real projective space $\mathbb{P}^n$. We take $n = 3$ and $m = 2$ in the most realistic situation.

The above problem can be decomposed into sub-problems at various levels (cf. [HZ03]). We start from an ideal setting in the middle of the whole reconstruction procedure, where a sufficiently large number of point correspondences across the r images are known without error in advance. Such point correspondences are regarded as points in the image of the following (rational) map combining r-tuple of cameras,

$$\phi \coloneqq (\varphi_1, \ldots, \varphi_r)\colon \mathbb{P}^n \dashrightarrow \prod_{j=1}^{r} \mathbb{P}^{m_j}. \tag{2}$$

It means that we have complete knowledge of the image of ϕ from the beginning, whereas ϕ is unknown. The image of ϕ, denoted by X_ϕ, is called a *multi-view variety*. It is often singular, but have nice properties, enabling the reconstruction procedure.

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The direct problem $\phi \mapsto X_\phi$ is just taking the image of ϕ, and hence, is almost trivially well-posed (in the sense of Hadamard). In [IMU20], we see that the inverse problem $X_\phi \mapsto \phi$ is also well-posed (up to projective transformations) if the configuration is general, $\sum_{j=1}^{r} m_j > n$ and $\mathbf{m} \neq (1^{n+1}) \coloneqq (1, \dots, 1)$.

This kind of inverse problem is often referred to as the projective reconstruction problem in computer vision. After introducing our result for projective reconstruction, I plan to discuss the similar problems (e.g., affine and metric reconstruction problems) and the relation with the problems that require more analysis, such as tomography.

References

[HZ03] Richard Hartley and Andrew Zisserman, *Multiple view geometry in computer vision*, second ed., Cambridge University Press, Cambridge, 2003, With a foreword by Olivier Faugeras. MR 2059248

[IMU20] Atsushi Ito, Makoto Miura, and Kazushi Ueda, *Projective reconstruction in algebraic vision*, Canad. Math. Bull. **63** (2020), no. 3, 592–609. MR 4148121