Inverse problems and theory of reproducing kernels – theory (abstract)

Saburou Saitoh
Institute of Reproducing Kernels
saburou.saitoh@gmail.com

December 8, 2020

1 Introduction

At least, until about 30 years ago, we had very difficult inverse problems that are important in many practical problems (fundamentals) as follows:

1): Inverse source problem; that is in the Poisson equation

$$\triangle u = -\rho,$$

from the observation of the potential u for the out side of the support ρ, look for the source ρ.

2): The problem in the heat conduction; that is, from some heat $u(x,t)$ observation at a time t, look for the initial heat $u(x,0)$.

3): Real inversion formulas for the Laplace transform.

These problems were indeed difficult in both mathematics and numerical realizations of the solutions and so, they are called ill-posed problems and very famous difficult problems.

We were able to solve these problems by using the theory of reproducing kernels applying the Tikhonov regularization. However, for the real inversion formula of Laplace transform, we needed the great power of computers by H. Fujiwara.

For any practical numerical analysis, the important problem is on the discretization procedure for analytical inverse problem solutions. At this very important point, we will see that the theory of reproducing kernels is very good mathematics. These global theories were published in the book [22] and our method is applicable in some general problems in the viewpoint of practical problems. Here, we state their essential parts. Here, we will state its theoritical parts and Professor T. Matsuura will give their numerical examples.

2 Inversion Formulas

3 Best Approximations, as a connection

4 The Tikhonov Regularization

5 Real and Numerical Inversions of the Laplace Transform

6 The Aveiro Discretization Method

7 A Typical Example of the Aveiro Discretization Method With ODE

References

[1] N. Aronszajn, Theory of reproducing kernels. Trans. Amer. Math. Soc., **68**(1950), 337–404.

[2] L. P. Castro, H. Fujiwara, S, Saitoh, Y. Sawano, A. Yamada and M. Yamada, Fundamental error estimates inequalities for the Tikhonov regularization using reproducing kernels. International Series of Numerical Mathematics **161**(2010), Inequalities and Applications 2010, Springer, Basel: 87-101

[3] L. P. Castro, H. Fujiwara, M. M. Rodrigues, S. Saitoh and V. K. Tuan, Aveiro Discretization Method in Mathematics: A New Discretization Principle, MATHEMATICS WITHOUT BOUNDARIES: SURVEYS IN PURE MATHEMATICS, Edited by Panos Pardalos and Themistocles M. Rassias 37-92 Springer (2014).

[4] L. P. Castro, H. Fujiwara, T. Qian and S. Saitoh, How to catch smoothing properties and analyticity of functions by computers? MATHEMATICS WITHOUT BOUNDARIES: SURVEYS IN INTERDISCIPLINARY RESEARCH, Edited by Panos Pardalos and Themistocles M. Rassias 101–116 Springer (2014).

[5] H. Fujiwara, Applications of reproducing kernel spaces to real inversions of the Laplace transform. RIMS Koukyuuroku 1618 (2008), 188–209.

[6] H. Fujiwara, T. Matsuura, S. Saitoh and Y. Sawano, Numerical real inversion of the Laplace transform by using a high-accuracy numerical method. Further Progress in Analysis: 574–583 World Sci. Publ., Hackensack, NJ (2009).

[7] H. Fujiwara, Numerical real inversion of the Laplace transform by reproducing kernel and multiple-precision arithmetic. Progress in Analysis and its Applications, Proceedings of the 7th International ISAAC Congress: 289–295 World Scientific (2010).

[8] H. Fujiwara and N. Higashimori, Numerical inversion of the Laplace transform by using multiple-precision arithmetic. Libertas Mathematica (new series), **34**(2014), No. 2, 5–21.

[9] H. Fujiwara and S. Saitoh, The general sampling theory by using reproducing kernels. CONTRIBUTIONS IN MATHEMATICS AND ENGINEERING In Honor of Constantin Caratheodory eds. Panos Pardalos and Th. M. Rassias, Springer (2010).

[10] P. C. Hansen, Analysis of discrete ill-posed problems by means of the L-curve. SIAM REVIEW, 34 (1992), 561-580.

[11] C. L. Lawson and R. J. Hanson, Solving least squares problems. Prentice-Hall, Englewood Cliffs. (1974).

[12] A. N. Kolmogoroff, Stationary sequences in Hilbert's space. Bolletin Moskovskogo Gosudarstvenogo Universiteta, Matematika (1941) 2: 40pp. (in Russian)

[13] T. Matsuura and S. Saitoh, Analytical and numerical inversion formulas in the Gaussian convolution by using the Paley-Wiener spaces. Applicable Analysis, **85** (2006), 901–915.

[14] T. Matsuura and S. Saitoh, General integral transforms by the concept of generalized reproducing kernels, P. Dan et al (eds.), New Trends in Analysis and Interdisciplinary Applications. Trends in Mathematics, Birkhäuser (2017), 379–386.

[15] Y. Mo and T. Qian, Support vector machine adapted Tikhonov regularization method to solve Dirichlet problem. Appl. Math. Comput. **245**(2014), 509–519.

[16] E. M. Rocha, A reproducing kernel Hilbert discretization method for linear PDEs with nonlinear right-hand side. Libertas Mathematica (new series), **34**(2014), no. 2, 91-104.

[17] S. Saitoh, *Hilbert spaces induced by Hilbert space valued functions,* Proc. Amer. Math. Soc., **89** (1983), 74–78.

[18] S. Saitoh, Integral Transforms, Reproducing Kernels and their Applications. Pitman Res. Notes in Math. Series 369 (1997), Addison Wesley Longman, Harlow, CRC Press, Taylor & Francis Group, Boca Raton London, New York (in hard cover).

[19] S. Saitoh, Various operators in Hilbert space induced by transforms. International J. of Applied Math. 1 (1999), 111–126.

[20] S. Saitoh, Theory of reproducing kernels: Applications to approximate solutions of bounded linear operator equations on Hilbert spaces. Amer. Math. Soc. Transl. Ser. 2: 230 (2010), 107–137.

[21] S. Saitoh and Y. Sawano, Generalized reproducing kernels and generalized delta functions, P. Dan et al (eds.), New Trends in Analysis and Interdisciplinary Applications. Trends in Mathematics, Birkhäuser (2017), 395–400.

[22] S. Saitoh and Y. Sawano, Theory of Reproducing Kernels and Applications, Developments in Mathematics **44**, Springer (2016).