Numerical homogenization of dual-phase steel by nonlinear conjugate gradient method.

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Abstract

The numerical homogenization method is a numerical method for estimating the average properties of materials by numerically solving partial differential equations with periodic boundary conditions defined in a rectangular region (cell problem). In this talk, we propose a numerical method to efficiently solve the cell problem that appears in the numerical homogenization method for nonlinear composites that follow the elasto-plastic constitutive law (Hencky's model) based on the total strain theory:

$$\begin{aligned}\sigma &= C_0 : \epsilon^e \\ \epsilon_{ij} &= \epsilon^e_{ij} + \epsilon^p_{ij} \\ \epsilon^p_{ij} &= \Lambda(|\sigma^D|)\sigma^D_{ij}.\end{aligned}$$

The function Λ models the plastic behavior of the material and it can be experimentally estimated by uniaxial tensile test.

We consider a dual phase material (e.g., martensite and ferrite) a periodically distributed with a representative volume element V. The cell problem is given by the system

$$(P) \qquad \begin{cases} -\operatorname{div}\sigma = 0 \quad \text{in } V \\ \epsilon_{ij} + E_{ij} = \frac{1}{2G}s_{ij} + \frac{1}{9K}\operatorname{tr}(\sigma)\delta_{ij} + \Lambda|\sigma^D|s_{ij} \\ \epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right). \end{cases}$$

The solution of the system minimizes the functional

$$J(\boldsymbol{u}) = \int_V W(\epsilon(\boldsymbol{v}) + \boldsymbol{E})dV.$$

In this talk, we propose a numerical method for solving the minimization problem by nonlinear conjugate gradient method.