On an inverse Robin eigenvalue problem appearing in thin coating problems

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Let $\Omega \subset \mathbb{R}^n$ $(n \geq 2)$ be a bounded domain with boundary $\partial\Omega$ of class C^2, and γ, Γ_D be disjoint nonempty closed subsets of the boundary $\partial\Omega$ such that $\partial\Omega = \Gamma_D \cup \gamma$. Let $h \in C^0(\gamma)$ and $h > 0$. In this talk, we consider the following Robin eigenvalue problem:

$$\begin{cases} -\Delta u = \lambda u & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_D, \\ hu + \partial_\nu u = 0 & \text{on } \gamma, \end{cases}$$

where ν is the outward unit normal vector of $\partial\Omega$. We only consider the principal eigenvalue and eigenfunction, and assume that the principal eigenfunction is positive and it is normalized by

$$\int_\Omega |u|^2 \, dx = 1.$$

Our aim in this talk is to study an inverse problem of the Robin eigenvalue problem. In particular, we consider the recovery of an unknown Robin coefficient h defined in the inaccessible part γ of the boundary $\partial\Omega$, given the principal eigenvalue $\lambda(h)$ and the Neumann data $\partial_\nu u(h)|_{\Gamma_D}$ on the accessible part Γ_D.

The inverse problem appears in thin coating problems [1]. Physically speaking, it is closely related to the problem that we determine a thin insulator coating for a heat conductor by measurements of the first eigenvalue and the Neumann data of an accessible part of the boundary. In this talk, we prove the uniqueness of the inverse problem and establish the identification by using a Neumann tracking type functional. Moreover, we show numerical results by using the gradient descent method. This is joint work with Matteo Santacesaria (University of Genoa) and based on the paper [2].

References

[1] A. Friedman, Reinforcement of the principal eigenvalue of an elliptic operator, Arch. Rational Mech. Anal. **73** (1980), 1–17.

[2] M. Santacesaria, T. Yachimura, On an inverse Robin spectral problem, Inverse Problems **36** (2020), 075004.

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